Chapter 11-12 Frappy #1

For many years, the medically accepted practice of giving aid to a person experiencing a heart attack was to have the person who placed the emergency call administer chest compression (CC) plus standard mouth-to-mouth resuscitation (MMR) to the heart attack patient until the emergency response team arrived. However, some researchers believed that CC alone would be a more effective approach.

In the 1990s a study was conducted in Seattle in which 518 cases were randomly assigned to treatments: 278 to CC plus standard MMR and 240 to CC alone. A total of 64 patients survived the heart attack. 29 in the group receiving CC plus standard MMR, and 35 in the group receiving CC alone. A test of significance was conducted on the following hypotheses.

H₀: The survival rates for the two treatments are equal.
Hₐ: The treatment that uses CC alone produces a higher survival rate.

This test resulted in a p-value of 0.0761.

(a) Interpret what this p-value measures in the context of this study.

(b) Based on this p-value and study design, what conclusion should be drawn in the context of this study? Use a significance level of α = 0.05.

(c) Based on your conclusion in part (b), which type of error, Type I or Type II, could have been made? What is one potential consequence of this error?

This test resulted in a p-value of 0.0761.

(a) Interpret what this p-value measures in the context of this study.

The p-value of 0.0761 measures the probability of observing a difference between the two sample proportions as large as or larger than the one observed, if the survival rates for the two treatments (CC alone and CC + MMR) are in fact the same. Need all 3 parts for essentially correct.

Because the p-value of 0.0761 is greater than 0.05, the null hypothesis should not be rejected. That is, there is not sufficient evidence to conclude that the treatment “CC alone” produces a higher survival rate than the standard treatment “CC + MMR.” Need to state conclusion and how the p-value compares to alpha for essentially correct

Based on your conclusion in part (b), which type of error, Type I or Type II, could have been made? What is one potential consequence of this error?

Because the null hypothesis was not rejected, a Type II error could have occurred. A possible consequence is that CC + MMR would continue as the accepted practice when, in fact, CC alone would result in a higher survival rate. Need a correct type and consequence for essentially correct.

*Note: don't get carried away with consequences. (not everything leads to death!)
Chapter 11-12 Frappy #2

A researcher wants to conduct a study to test whether listening to soothing music for 20 minutes helps to reduce diastolic blood pressure in patients with high blood pressure, compared to simply sitting quietly in a noise-free environment for 20 minutes. One hundred patients with high blood pressure at a large medical clinic are available to participate in this study.

(a) Propose a design for this study to compare these two treatments.

Matched pairs design (each person gets both treatments)

100 patients

random assignment

Group 1
50 patients

Treatment
20 mins music

Treatment
20 mins quiet

Group 2
50 patients

Treatment
20 mins quiet

Treatment
20 mins music

allow time to pass

for random assignment: put numbers 1-100 into a hat and draw out 50 without replacement. These are group 1 and the rest are group 2

(b) The null hypothesis for this study is that there is no difference in the mean reduction of diastolic blood pressure for the two treatments and the alternative hypothesis is that the mean reduction in diastolic blood pressure is greater for the music treatment. If the null hypothesis is rejected, the clinic will offer this music therapy as a free service to their patients with high blood pressure. Describe Type I and Type II errors and the consequences of each in the context of this study, and discuss which one you think is more serious.

Type I error: Concluding that soothing music does reduce mean diastolic blood pressure compared to sitting quietly, when in fact it does not. The consequence of this type of error is that the clinic will offer music therapy when it is not effective.

Type II error: Soothing music does reduce diastolic blood pressure compared to sitting quietly, but we fail to detect this and conclude that it does not. The consequence of this type of error is that the clinic will choose not to offer music therapy when it would have been effective.

Which type of error is more serious? A case can be made for either type of error, and the student can take either side as long as a reasonable justification is given. For example, the student can say a Type I error is more serious because it will cost the clinic money with no benefit, or the student can say that a Type II error is more serious because the clinic will miss an opportunity to improve the health and well-being of its patients.
Because the sample size is small, the Normal curve is not a good approximation.

\[ \Phi(0.5) \neq 10 \]

The preference data for the 8 randomly selected consumers are given in the table below.

<table>
<thead>
<tr>
<th>Sampled Juice Preference</th>
<th>Citrus Fresh</th>
<th>Tropical Taste</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

Based on these preferences and your previous work, test the hypothesis in part (a).}

\[
p \neq 0.5\]

\[
\Phi(p^{0.5}) = 0.1445
\]

A significance level of .05 is insufficient to conclude that there exists a preference for either juice product.

Increase the sample size.

This will:

- decrease the variation which could make the same statistic significant.
- increase the power of the test
- reduce the probability of making a type II error.
Chapter 11-12 Frappy explained #4

Researchers at a large health maintenance organization (HMO) are planning a study of a certain mild illness. They will select a random sample of patients who are ages 35 to 54 and see if they contract the illness in the next year. The researchers are interested in estimating the proportions of men and of women who are likely to develop the illness in each of 4 age-groups: 35-39, 40-44, 45-49, and 50-54.

The researchers plan to include 2,000 patients in the study. Suppose the researchers draw a random sample from all of the patients at this HMO who are ages 35 to 54 and find the following numbers within each gender and age-group:

<table>
<thead>
<tr>
<th>Age-Group</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>35-39</td>
<td>350</td>
<td>445</td>
</tr>
<tr>
<td>40-44</td>
<td>230</td>
<td>370</td>
</tr>
<tr>
<td>45-49</td>
<td>150</td>
<td>245</td>
</tr>
<tr>
<td>50-54</td>
<td>60</td>
<td>150</td>
</tr>
</tbody>
</table>

(a) Suppose that at the end of the study, 10 percent of the females in the 40-44 age-group contracted the illness. Calculate a 95 percent confidence interval to estimate the population proportion of females in this age-group that contracted the illness.

Interpret this confidence interval in the context of this situation.

\[
\hat{p} = \frac{x}{n}
\]

\[
\hat{p} = 0.10, \quad n = 200
\]

\[
\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}
\]

\[
0.10 \pm 1.96 \sqrt{\frac{0.10(0.90)}{200}} = 0.10 \pm 0.0366
\]

\[
(0.063, 0.137)
\]

We are 95% confident that the true proportion of females between 40 and 44 years of age contracting the illness lies between 0.063 and 0.137.

(b) Suppose that at the end of the study, 10 percent of the males in the 40-44 age-group contracted the illness.

The corresponding 95 percent confidence interval to estimate the population proportion of males in this age-group that contracted the illness is (0.061, 0.139).

Note that this interval and the interval in part (a) are of different lengths even though the two sample proportions were identical. What would be an alternative way to allocate a sample of 2,000 subjects so that the 95 percent confidence interval widths for all male age-groups and for all female age-groups (i.e., for all 8 groups) would be the same when the sample proportions are the same? Justify your answer.

The width of the confidence interval depends on the sample size. The sample sizes need to be the same in order to have the 95% confidence intervals the same with for the same sample proportion. You could use a stratified sample design to randomly select 250 from each of the 8 groups.

(c) Based on previous studies, researchers believe that the percentages of those who contract the illness will be similar for males and females, and therefore plan to ignore gender when selecting a sample for this study. Previous studies also indicate that the percentages of adults who will contract this illness in the 35-39, 40-44, 45-49, and 50-54 age-groups are anticipated to be 5%, 8%, 20%, and 35%, respectively. How should the sample of 2,000 subjects be allocated with respect to age-groups so that the widths of the 95 percent confidence intervals for the four groups will be approximately the same? Justify your answer.

The length of a confidence interval is also dependent on the proportion. \( \hat{p} = \frac{x}{n} \) as \( \hat{p} \) increases, the margin of error increases. Therefore, if the proportion of successes increases as the age increases, the sample size should also increase in order to get lower the standard deviation of \( \hat{p} \).

The sample should be allocated as follows:

<table>
<thead>
<tr>
<th>Age-Group</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>35-39</td>
<td>350</td>
</tr>
<tr>
<td>40-44</td>
<td>400</td>
</tr>
<tr>
<td>45-49</td>
<td>600</td>
</tr>
<tr>
<td>50-54</td>
<td>850</td>
</tr>
</tbody>
</table>

Go on to the next page.
Chapter 11-12 Frappy #5

Some boxes of a certain brand of breakfast cereal include a voucher for a free video rental inside the box. The company that makes the cereal claims that a voucher can be found in 20 percent of the boxes. However, based on their experiences eating this cereal at home, a group of students believes that the proportion of boxes with vouchers is less than 0.2. This group of students purchased 65 boxes of the cereal to investigate the company's claim. The students found a total of 11 vouchers for free video rentals in the 65 boxes.

Suppose it is reasonable to assume that the 65 boxes purchased by the students are a random sample of all boxes of this cereal. Based on this sample, is there support for the students' belief that the proportion of boxes with vouchers is less than 0.2? Provide statistical evidence to support your answer.

\[ \hat{p} = \frac{x}{n} = \frac{11}{65} = 0.1692 \]
\[ \sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.20(1-0.20)}{65}} = 0.0466 \]

**Spec: 0.20**

**I will do a proportion z-test**

**Conditions**
- Assume sample of boxes is unbiased estimator of true proportion
- Assume population \( \geq 10n \rightarrow n \geq 650 \)
- \( n \hat{p} > 10? \Rightarrow \frac{n(1-p)}{10} \geq 10? \)
- \( n = 65, \quad n > 30 \)

- \( 1.584 \leq z \leq 2.96 \)

- Assume \( H_0 \) for sampling distribution

- \( z \) test stat = \[ \frac{\text{stat-parameter}}{\text{standard error}} = \frac{0.1692 - 0.20}{0.0466} = -0.6210 \]

\[ p(\hat{p} < 0.1692) = p(z < -0.6210) = \text{normalcdf}(-10^9, -0.6210) = 0.2673 \]

Assuming the proportion of vouchers found in boxes is 0.20, there is a 0.2673 chance of getting a sample with a proportion more extreme than 0.1692.

Large \( p \rightarrow \) Fail to reject \( H_0 \).

There is insufficient evidence to claim that the proportion of boxes with vouchers is less than 20 percent.
A pharmaceutical company has developed a new drug to reduce cholesterol. A regulatory agency will recommend the new drug for use if there is convincing evidence that the mean reduction in cholesterol level after one month of use is more than 20 milligrams (mg/dL), because a mean reduction of this magnitude is approx. normal by CLT.

A confidence interval is closely linked to a two-sided hypothesis test. If you took the adjusted p-value to a two-sided test, the p-value would be 0.066 which would not be convincing evidence. This would now agree with the conclusion of the confidence interval.

(b) Because the 95% confidence interval includes 20, the regulatory agency is not convinced that the new drug is better than the current best-seller. The pharmaceutical company tested the following hypotheses:

\[ H_0: \mu = 20 \] versus \[ H_1: \mu > 20. \]

where \( \mu \) represents the population mean reduction in cholesterol level for the new drug.

The test procedure resulted in a t-value of 1.89 and a p-value of 0.033. Because the p-value was less than 0.05, the company believes that there is convincing evidence that the mean reduction in cholesterol level for the new drug is more than 20. Explain why the confidence interval and the hypothesis test led to different conclusions.

A confidence interval is closely linked to a two-sided hypothesis test. If you took the adjusted p-value to a two-sided test, the p-value would be 0.066 which would not be convincing evidence. This would now agree with the conclusion of the confidence interval.

(c) The company would like to determine a value \( L \) that would allow them to make the following statement.

We are 95% confident that the true mean reduction in cholesterol level is greater than \( L \).

A statement of this form is called a one-sided confidence interval. The value of \( L \) can be found using the following formula.

\[ L = \bar{x} - t^* \frac{s}{\sqrt{n}} \]

This has the same form as the lower endpoint of the confidence interval in part (a), but requires a different critical value, \( t^* \). What value should be used for \( t^* \)?

\[ \text{df} = 49 \] (or 50 now)

use a tail probability of 0.05 instead of 0.025 because there

only one tail and we still need a 0.05 significance level.

\[ 50 - t^* = 1.676 \]

Recall that the sample mean reduction in cholesterol level and standard deviation are 24 mg/dL and 15 mg/dL, respectively. Compute the value of \( L \).

\[ L = 24 - 1.676 \left( \frac{15}{\sqrt{50}} \right) \]
\[ = 24 - 3.558 \]
\[ L = 20.442 \text{ mg/dL} \]

We are 95% confident that the true mean reduction in cholesterol level is greater than 20.4 mg/dL.

(d) If the regulatory agency had used the one-sided confidence interval in part (c) rather than the interval constructed in part (a), would it have reached a different conclusion? Explain.

Yes, it would have reached a different conclusion.

The regulatory agency said it would recommend the new drug for use if there was convincing evidence that the mean reduction in cholesterol level after one month of use was 20 mg/dL. They were not convinced that the new drug was better than the current best-seller because the confidence interval did not contain 20 mg/dL. However, the new interval proves convincing evidence at the 0.05 level that the reduction is greater than 20 mg/dL, so the regulatory agency would probably have reached the different conclusion that they would recommend the new drug for use.
Chapter 11-12 Frappy #7

When a law firm represents a group of people in a class action lawsuit and wins that lawsuit, the firm receives a percentage of the group’s monetary settlement. That settlement amount is based on the total number of people in the group—the larger the group and the larger the settlement, the more money the firm will receive.

A law firm is trying to decide whether to represent car owners in a class action lawsuit against the manufacturer of a certain make and model for a particular defect. If 5 percent or less of the cars of this make and model have the defect, the firm will not recover its expenses. Therefore, the firm will handle the lawsuit only if it is convinced that more than 5 percent of cars of this make and model have the defect. The firm plans to take a random sample of 1,000 people who bought this car and ask them if they experienced this defect in their cars.

(a) Define the parameter of interest and state the null and alternative hypotheses that the law firm should test.

\[ p = \text{percent of cars of this make and model that have the defect} \]

\[ H_0: p=0.05 \]
\[ H_a: p>0.05 \]

(b) In the context of this situation, describe Type I and Type II errors and describe the consequences of each of these for the law firm.

Type I error would occur if the firm takes the lawsuit when in fact only 5 percent of the cars have the defect. The consequences of this error would be the firm would incur more expenses than they collect from the manufacturer.

Type II error would occur if the firm does not take the lawsuit when in fact more than 5 percent of the cars have the defect. The consequences of this error would be the firm misses out on potential profit from a large lawsuit.
Chapter 11-12 Frappy #8

A growing number of employers are trying to hold down the costs that they pay for medical insurance for their employees. As part of this effort, many medical insurance companies are now requiring clients to use generic brand medicines when filling prescriptions. An independent consumer advocacy group wanted to determine if there was a difference, in milligrams, in the amount of active ingredient between a certain “name” brand drug and its generic counterpart. Pharmacies may store drugs under different conditions. Therefore, the consumer group randomly selected ten different pharmacies in a large city and filled two prescriptions at each of these pharmacies, one for the “name” brand and the other for the generic brand of the drug. The consumer group’s laboratory then tested a randomly selected pill from each prescription to determine the amount of active ingredient in the pill. The results are given in the following table.

### ACTIVE INGREDIENT

( in milligrams)

<table>
<thead>
<tr>
<th>Pharmacy</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name brand</td>
<td>245</td>
<td>244</td>
<td>240</td>
<td>250</td>
<td>243</td>
<td>246</td>
<td>246</td>
<td>246</td>
<td>247</td>
<td>250</td>
</tr>
<tr>
<td>Generic brand</td>
<td>246</td>
<td>240</td>
<td>235</td>
<td>237</td>
<td>243</td>
<td>239</td>
<td>241</td>
<td>238</td>
<td>238</td>
<td>234</td>
</tr>
</tbody>
</table>

Based on these results, what should the consumer group's laboratory report about the difference in the active ingredient in the two brands of pills? Give appropriate statistical evidence to support your response.

**Matched Pair T-test**

**Assumptions**
- This was a SRS.
- I don't know if the population is normal, but since n is medium (6) and I don't see any outliers in the data, it is relatively normal, I will continue with the test.

**Ho:** $\mu_{name} = \mu_{generic}$

**Ha:** $\mu_{name} \neq \mu_{generic}$

\[ \bar{x} = 6.6, \quad S_x = 5.275, \quad n = 10, \quad \sigma = 1.648 \]

**p-value = 0.033**

\[ x = 0.05 \]

If I assume that the mean of differences between the name brand and generic drug is 0, there is a 0.33% chance of seeing my results. This is low, so I have reason to doubt that the mean of difference between the name brand and generic drug is 0. This means that it does not appear that the two types of drugs contain the same amount of active ingredient. I would suggest the consumer group's laboratory to report that there is no difference in the active ingredient amounts.